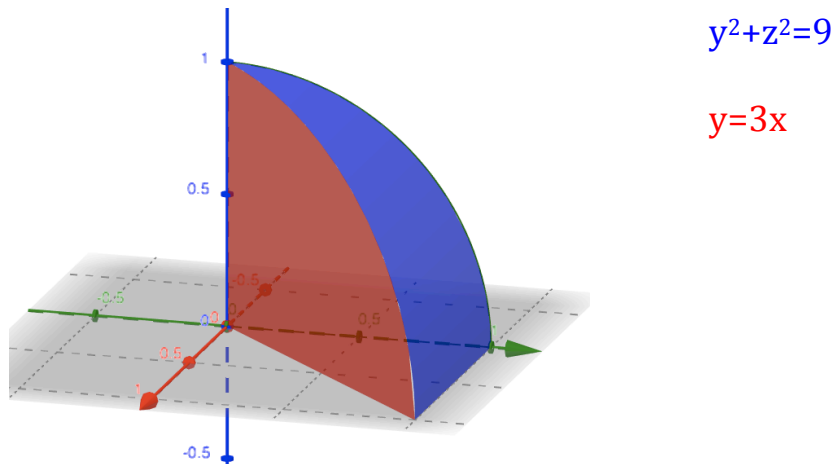


Triple Integral Example:

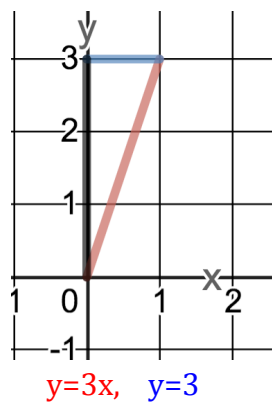
Compute $\iiint_E \left(\frac{1}{9}-z\right) dV$ where E is the solid bound by $\begin{cases} y^2+z^2=9 \\ y=3x \end{cases}$ in the first octant.

Sketch E: Link to graph on 5C page: <https://www.geogebra.org/m/v8tJbE3j> (Scale off here)



dz first

D is projection in xy plane:



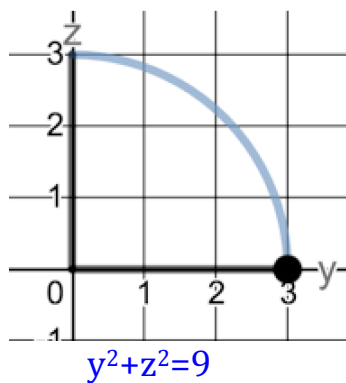
$$\int_0^1 \int_{3x}^3 \int_0^{\sqrt{9-y^2}} \left(\frac{1}{9}-z\right) dz dy dx$$

$$\int_0^3 \int_0^{\frac{y}{3}} \int_0^{\sqrt{9-y^2}} \left(\frac{1}{9}-z\right) dz dx dy$$

Answer: -73/24

dx first

D is proj in yz plane:

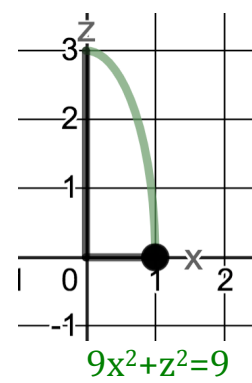


$$\int_0^3 \int_0^{\sqrt{9-y^2}} \int_0^{\frac{y}{3}} \left(\frac{1}{9}-z\right) dx dz dy$$

$$\int_0^3 \int_0^{\sqrt{9-z^2}} \int_0^{\frac{y}{3}} \left(\frac{1}{9}-z\right) dx dy dz$$

dy first

D is proj in xz plane:



$$\int_0^1 \int_0^{\sqrt{9-9x^2}} \int_0^{\sqrt{9-z^2}} \left(\frac{1}{9}-z\right) dy dz dx$$

$$\int_0^3 \int_0^{\sqrt{1-\frac{1}{9}z^2}} \int_0^{\sqrt{9-z^2}} \left(\frac{1}{9}-z\right) dy dx dz$$